DISTRIBUTION OF THE CIRCULATION TIMES IN A BATCH MIXED BY TWO IMPELLERS ON A COMMON SHAFT*

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Received April 27, 1988
Accepted June 14, 1988

The paper has been devoted to the study of the circulation times in a batch mixed by several impellers on a common shaft. A simple method has been used to carry out the measurements based on visual observation of the motion of a tracer particle in the mixed batch. Individual passages of the tracer particle through the check surface have been signalized to a computer performing the primary processing of data, calculation of the basic statistical characteristics (expected value and variance) and the frequency function of the circulation times of the tracer particle. A model developed earlier for a vessel mixed by a single impeller has been used to interpret the data. This theoretical model has been compared with experimental results.

Vessels where the batch is mixed by several impellers on a common shaft are even more popular in industry. The diameter of these vessels is almost in all cases several times smaller than the height of the liquid in the vessel. The hydrodynamics of these systems is more complex than those mixed by a single impeller as it represents the effects of several flows induced by individual impellers.

A simple method of following a tracer particle has been used to examine the flow of fluids in these systems while keeping the density of the particle approximately equal to the density of the liquid. The tracer particle thus approximately follows the streamlines in the mixed batch. This way we can therefore examine the distribution of the residence times of the tracer particle in individual parts of the mixed batch, or the distribution of the circulation times, i.e. the distribution of the times required for a repeated passage through the rotor region of the impeller. From thus found distributions one can derive also the frequency functions of the circulation times. These functions permit one to estimate the structure of the flow of liquid in the mixed batch, the effect of the geometry parameters in the system and the contribution of of individual impellers to the net flow in the batch.

* Part LXXV in the series Studies on Mixing; Part LXXIV Collect. Czech. Chem. Commun. 53, 1198 (1988).

For systems with a single impeller there exist a number of papers describing the use of this method for the estimation of the distribution of the circulation times in the vessel. Porcelli and Marr[1] and Marr and Johnson[2], for instance, investigated the effect of the geometry and physical parameters on the mean circulation time defined by

$$\Delta t = T/m\,, \tag{1}$$

where T is the total time of duration of the experiment and m is the number of passages of the tracer particle through the rotor region. The frequency of the circulation times, however, has been described neither quantitatively nor qualitatively. Fořt[3], who starts from the above cited papers, has used the following relation to describe the frequency function

$$f(\Delta t) = k_1 \exp\left(-k_2\, \Delta t\right), \tag{2}$$

where the constants k_1 and k_2 depend on the physical and geometry parameters of the system. For the mixed batch he used the model of a continuous system where the inlet is identical with the outlet.

Also Inoue and Sato[4] measured the distribution of the circulation times in the batch mixed by the turbine impeller. The tracer particle in their case was a cotton thread. The authors claim that the frequency of revolution of the impeller has no effect on the shape of the time distribution which is affected only by the location of the impeller in the vessel and the height of the liquid. They derive also the relationship between the distribution of the circulation times and the course of homogenation in the vessel. Nevertheless, the analysis of the found relationships has not been attempted.

The frequency functions and the circulation time distribution has been investigated by Kudrna et al.[5]. These authors compared the experimentally found frequency function with the theoretical function following from the concept of a random motion of a tracer particle in the mixed batch under some simplifying assumptions

$$f(\Delta t) = \frac{2\omega r}{\sqrt{\pi}} \exp\left[-r^2 \frac{\exp\left(-2\omega\, \Delta t\right)}{1 - \exp\left(-2\omega\, \Delta t\right)}\right] \frac{\exp\left(-\omega\, \Delta t\right)}{\left(1 - \exp\left(-2\omega\, \Delta t\right)\right)^{3/2}}\,. \tag{3}$$

The turbine impeller and the impeller with inclined blades were used for experiments. It was proven that the dependence (3) describes well the experimentally obtained frequency functions[6]. Also the physical meaning of the parameters ω and r in Eq. (3) was confirmed.

Mukataka, Kataoka and Takahashi[7] investigated the distribution of the circulation times in the batch mixed by a single impeller in a non-Newtonian batch, in some cases with aeration. The tracer particle was a natural magnet in a plastic capsule. Passages of this particle through a control surface were monitored by coils in the plane of the impeller. The authors considered the distribution of the residence times to be lognormal, this fact being proven by plotting experimental data on the probability paper. The paper also discusses the effect of rheological properties of the batch on the circulation times of the tracer particle. In their next paper[8] the same authors investigated the distribution of the circulation times in a batch mixed by two impellers on a common shaft while the method used as well as the physical conditions of the experiments remained the same. The authors considered the distribution of the circulation times to be again lognormal in region of both the upper and the lower impeller and claim that both distributions are identical.

Funahashi, Harada and Taguchi[9] examined the distribution of the circulation times in a non-Newtonian pseudoplastic liquid by a single turbine with six blades. The method used for the measurement of the distribution of the circulation times is similar to that used in the previous paper. The authors found that the frequency function exhibited for lower frequencies of revolution of the impeller bimodal character, which, for increasing intensity of mixing, tended to a frequency function with a single maximum. The authors tried to explain this fact by the existence of three regions in the mixed batch (the region of micro- and macroflow and the stagnating region), whose size varied with the change of the intensity of mixing.

Nienow and Kuboi[10] used a system of two impellers on a single shaft to mix non-Newtonian solutions of carboxymethylcellulose in water. The lower impeller was a turbine; the upper an impeller with inclined blades. The method of measurements consisted in recording the motion of polystyrene spheres by a video camera. The playback served to examine the distribution of the circulation times. Primary interest was in the observation of the exchange stream between the mixed regions. The authors did not deal directly with the frequency functions, instead they computed only the probability of the transition of a particle from one region into another.

Fořt et al.[11] investigated the effect of the distance between the impellers on the distribution of the circulation times. The paper studies experimentally the circulation in a batch mixed by either two turbines or a turbine and a paddle impeller. The mixed liquid was a water solution of NaCl. The experimental data served to compute the flow criteria and to examine the effect of the geometry and physical parameters of the system on the distribution of the circulation times for individual impellers.

THEORETICAL

For preliminary interpretation of experimental data from the standpoint of the description of the distribution of the circulation times the following relationship was proposed. This relationship had been used successfully earlier for the same purpose and the standard configuration with a single impeller (see Eq. (*3*)). This equation can be written more concisely in the form

$$f(\Delta t) = \frac{2r}{\sqrt{\pi}} \exp\left[-r^2 h^2(\Delta t)\right] g(\Delta t)\,, \tag{4}$$

where

$$h(\Delta t) \equiv \frac{\exp(-\omega \Delta t)}{(1 - \exp(-2\omega\Delta t))^{1/2}} \tag{4a}$$

and

$$g(\Delta t) \equiv \frac{\mathrm{d}h(\Delta t)}{\mathrm{d}\Delta t} = \frac{\omega \exp(-\omega\Delta t)}{(1 - \exp(-2\omega\,\Delta t))^{3/2}}\,. \tag{4b}$$

In these relationships Δt designates the circulation time; ω and r are parameters of the distribution. For a statistical estimate of the values of these parameters the maximum likelihood method was employed (see e.g. ref.[12]). The following likelihood function was defined:

$$P(\omega, r) = \ln \prod_{i=1}^{n} f(\Delta t_i \mid \omega, r)\,, \tag{5}$$

where Δt_i designated individual values of the statistical set. The estimate of the parameters is determined from the maximum of this function.

After substituting into Eq. (5) from Eqs (4) and after appropriate differentiations we obtain

$$\frac{\partial P}{\partial \omega} = \frac{m}{\omega} + \sum_{i=1}^{m} [2r^2 h(\Delta t_i)\, g(\Delta t_i)/\omega - 3h^2(\Delta t_i) - 1]\, \Delta t_i = 0 \tag{6}$$

$$\frac{\partial P}{\partial r} = \frac{m}{r} - 2r \sum_{i=1}^{m} h^2(\Delta t_i) = 0\,. \tag{7}$$

The parameter r can be directly determined from the last equation and substituted into Eq. (6). The result is an implicit relationship for the determination of the parameter omega.

$$\frac{1}{m} \sum_{i=1}^{m} \omega \Delta t_i \left[1 + 3h^2(\Delta t_i) - \frac{m}{\omega}\, h(\Delta t_i)\, g(\Delta t_i) / \sum_{i=1}^{n} h^2(\Delta t_i) \right] = 1\,. \tag{8}$$

From this equation one can obtain omega using the Newton iteration method[13]. Its initial estimate for this method was obtained on the basis of the fact that all moments of the function $f(\Delta t)$ are indirectly proportional to appropriate powers of this parameter. In the expression for the n-th moment

$$M_n = \int_0^{\infty} (\Delta t)^n f(\Delta t)\, \Delta t \tag{9}$$

one can clearly substitute $\omega\, \Delta t = p$ and upon considering Eqs (4) and (4a) write

$$\omega^n M_n = \frac{2r}{\sqrt{\pi}} \int_0^{\infty} p^n \exp\left(-r^2 h^2(p)\right) \frac{\mathrm{d}h}{\mathrm{d}p}\, \mathrm{d}p = \varphi_n(r)\,. \tag{10}$$

Thus modified integral is only a function of the parameter r. It can be easily proven that $M_0 = 1$. Values of the following moments have to be computed numerically. For an estimate of the parameters it suffices to evaluate the first two moments. By combining the first two moments one can eliminate the parameter omega to find the value of r:

$$M_1^2/M_2 = \varphi_1^2/\varphi_2 = \varphi(r) \tag{11}$$

and from the expression for the first moment then the value of omega. This approach permitted the determination of the sufficiently accurate initial estimate of both parameters.

EXPERIMENTAL

All measurements were carried out in a cylindrical vessel of internal dimeter $D = 0{\cdot}29$ m and the height of liquid $H = 2D$ (see Fig. 1). The vessel was equipped with four baffles of width $b = 0{\cdot}1D$ located symmetrically around the internal circumference of the cylinder. The vessel proper was located in another container of square cross section while all parts of both vessels were manufactured of perspex glass. The external vessel served for thermostating the mixed batch and was filled with distilled water.

In the center of the whole system on a common shaft there were two impellers of diameter $d = 0{\cdot}097$ m. The distance between the impellers and between the lower impellers and the bottom of the vessel could be arbitrarily altered. As impellers served either two turbine impellers or two impellers with inclined blades, or, eventually, their combination.

The measurements were carried out in two batches: A water solution of NaCl ($\varrho = 1\,040\ \mathrm{kg/m^3}$; $\eta = 0{\cdot}001$ Pa s) and the solution of NaCl in 40% glycerol ($\varrho = 1\,080\ \mathrm{kg/m^3}$; $\eta = 0{\cdot}00375$ Pa s). The range of frequency of revolution of the impeller was between $2{\cdot}5\ \mathrm{s^{-1}}$ and $7{\cdot}5\ \mathrm{s^{-1}}$. Corresponding range of the Reynolds numbers was between 7 000 and 70 000. The frequency functions were measured in dependence on the frequency of revolution of the impeller and the distance between the impellers ranging between 0·29 and 0·05 m.

The circulation times were observed visually by means of a tracer particle assembled of three discs manufactured of alkali polyamide and soldered perpendicularly one to another. The diameter of the tracer particle was approximately 0·006 m and its density was adjusted by a suspension of powdered copper in epoxy resin to the density of the solution. The time instances of the passages were recorded directly in the memory of an IQ-151 computer.

More detailed description of the experimental arrangement as well as the measuring method is given in the previous work[11].

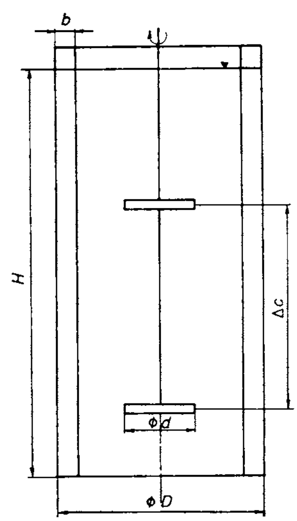

FIG. 1
Geometrical configuration of mixing equipment

RESULTS

Primary processing of the experimental data was based on the fact that the experimental set-up permitted us to distinguish and record in the computer memory the time instant of the passages through the rotor region of the upper and lower impeller and the time intervals between these events. These impulses were recorded on the time axis with a discrete division down to $\delta = 20$ ms. The result of the recording may be regarded as a function $I(j\delta)$ of a discrete argument, where j designates a non-negative integer. As an event we have defined such an instant on the time axis prior to which the function I just assumed a significant value, i.e. for the validity of the relations

$$t^{\mathrm{I}} = j\delta: \ [I(j\delta) < -q] \cap [I((j-1)\,\delta) > -q] \tag{12a}$$

$$t^{\mathrm{II}} = j\delta: \ [I(j\delta) > q] \cap [I((j-1)\,\delta) < q]\,. \tag{12b}$$

In these relationships the constant q designates a preset limit of resolution of the values of the function I. The probability of a very rapid variation of the function I from significantly positive to significantly negative value is taken to be zero.

In the same manner we have also determined the beginning of the discrete time axis: From the real beginning of the experiment we have monitored the course of the function I and upon the first occurence of validity of Eqs (12a) or (12b) the number j was assigned a zero value.

It is apparent that the instants of the change are of two types, corresponding to the change of the function I from a zero value to a significantly positive or significantly negative value. This physically characterises the instant when the particle enters the rotor region of the upper or the lower impeller and is distinguished by two values of the superscript $\alpha = (\mathrm{I}, \mathrm{II})$.

We shall now order the instant of changes into a sequence of increasing values regardless of the kind of the event

$$0 = t_0^{\alpha} < t_1^{\alpha} < \ldots < t_j^{\alpha} < t_{j+1}^{\alpha} < \ldots < t_m^{\alpha} = T\,,$$

where T designates the duration of the experiment. This sequence permits us then to define the circulation time as a time interval between two consecutive instants of the change

$$\Delta t_j^{\beta} = t_j^{\alpha} - t_{j-1}^{\alpha}\,, \tag{13}$$

while it is apparent that two values of the superscript alpha define four values of the superscript beta:

$$\Delta t_j^{\mathrm{I-I}} = t_j^{\mathrm{I}} - t_{j-1}^{\mathrm{I}}\,, \tag{13a}$$

this physically represents the time of circulation in the bottom impleller;

$$\Delta t_j^{\mathrm{II-II}} = t_j^{\mathrm{II}} - t_{j-1}^{\mathrm{II}} \tag{13b}$$

designates the circulation time in the upper impeller.

$$\Delta t_j^{\mathrm{I-II}} = t_j^{\mathrm{II}} - t_{j-1}^{\mathrm{I}} \tag{13c}$$

defines the time of circulation (transition) from the bottom to the upper impeller and

$$\Delta t_j^{\mathrm{II-I}} = t_j^{\mathrm{I}} - t_{j-1}^{\mathrm{II}} \tag{13d}$$

the circulation (transition) time in the opposite direction. These definitions allow classification of the set of the circulation times to four subsets whose elements are mutually disjunct.

Since the motion of the tracer particle is random owing to the turbulent flow of the batch the circulation times Δt^β are generally random quantities which must be processed according to the rules of mathematical statistics. A program has been therefore written allowing for each subset of experimental circulation times Δt^β the following data to be evaluated:

1. The number of data in each subset m^β, while we have clearly the condition $\sum_{\beta=1}^{4} m^\beta = m$, where m is the total number of measurements in the given series, i.e. the total number of passages of the tracer particle through all rotor regions.
2. The mean circulation time in each subset $\langle \Delta t^\beta \rangle$.
3. The standard deviation of the circulation times in each subset s^β.

For each subset the program further permitted the evaluation of an empirical distribution of the circulation times in the form of absolute frequencies. The width of the class interval and the class characteristics of individual classes were determined in a manner ensuring an easy comparison between results of various experiments.

The process of classification starts from the knowledge of the mean value $\langle \Delta t^\beta \rangle$ and the standard deviation s^β. Further the structure of the classes should satisfy the following requirements:

1. The distribution should have $L + 1$ classes designated by the indices $k = 0, 1, \ldots$ $\ldots, L$.
2. The mean value $\langle \Delta t^\beta \rangle$ should fail into the middle class.
3. The peripheral classes with indices $k = 0$ and $k = L$ shall encompass all outlying (i.e. very small or very large) values of the circulation times. Intermediate classes for $k = 1, \ldots, L - 1$ contain the remaining values for which it holds that

$\Delta t^\beta \in [\langle \Delta t^\beta \rangle - U s^\beta; \langle \Delta t^\beta \rangle + U s^\beta]$. The number U, characterising the width of the confidence interval, is suitably selected.

4. The width of the class interval V has a value which is assigned only one significant digit of the three following digits: 1, 2, and 5. (Thus, for instance $V = 0{\cdot}002$ or $V = 50$, but not $V = 51$.)

5. The beginning of the first class interval W is an integer multiple of the width of the class interval V. The requirements 4. and 5. are inconsistent with the requirement 3. for an arbitrary U. However, only fulfillment of the reqirements 4. and 5. provides results of different experiments in a comparable form.

The method proper after classification is as follows:

1. The width of the class interval V' is computed satisfying accurately the requirements 1. and 3.:

$$V' = 2U\, s^\beta/(L - 1) \tag{14}$$

2. The exponent E and mantissa N' of the semi-logarithmic expression of V' are determined:

$$E = \mathrm{int}\,(\log_{10} V')\,, \tag{15}$$

$$N' = V'/10^E\,. \tag{16}$$

The function int (x) is defined as an integer nearest from the left to x. If x is an integer then $\mathrm{int}\,(x) = x$. Mantisse N' computed from Eq. (*16*) lies in the interval $[1, 10)$.

3. Mantisse N of the sought width is determined from the relations

$$N = \left\{\begin{array}{l} 10\ [N' > \sqrt{62{\cdot}5}] \\ \ 5\ [\sqrt{62{\cdot}5} \geqq N' > \sqrt{10}] \\ \ 2\ [\sqrt{10} \geqq N' > \sqrt{1{\cdot}6}] \\ \ 1\ [\sqrt{1{\cdot}6} \geqq N'] \end{array}\right\}. \tag{17}$$

This choice of N ensures that the ratio between the width of the class V', determined from the conditions 1. and 3., and width of the class V really chosen, falls always into the following limits

$$V'/V \in \langle 0{\cdot}632;\ 1{\cdot}58 \rangle\,. \tag{18}$$

The selected class interval V thus cannot differ from the "theoretical" V more than as given by Eq. (*18*).

4. The width of the class interval V is computed from

$$V = N\,.\,10^E\,. \tag{19}$$

5. The beginning of the class interval is determined from

$$W = V(\mathrm{int}\,(\langle \Delta t^{\beta} \rangle / V - L/2) + 1/2)\,. \tag{20}$$

The program assigned individual values of the circulation times into the set up classes. Thus obtained absolute frequencies of the frequency function were smoothed by the method of orthogonal polynomials[14] and the approach described in the part Theoretical served to determine the parameters of the distribution. The theoretical frequency function, given by Eq. (*4*), was compared with the empirical one by means of the Pearson test on a 5% significance level. Example of graphical comparison are given in Fig. 2.

DISCUSSON

The principal goal of this work was experimental determination of the distribution of the circulation times in batch mixed by two rotating impellers mounted on a common shaft. The measuring method, developped for this purpose, proved to be sufficiently reliable and accurate with rapid processing of primary results. Thanks to this approach we were able to obtain more than 350 experimental estimates of the frequency functions for various conditions and geometrical configurations.

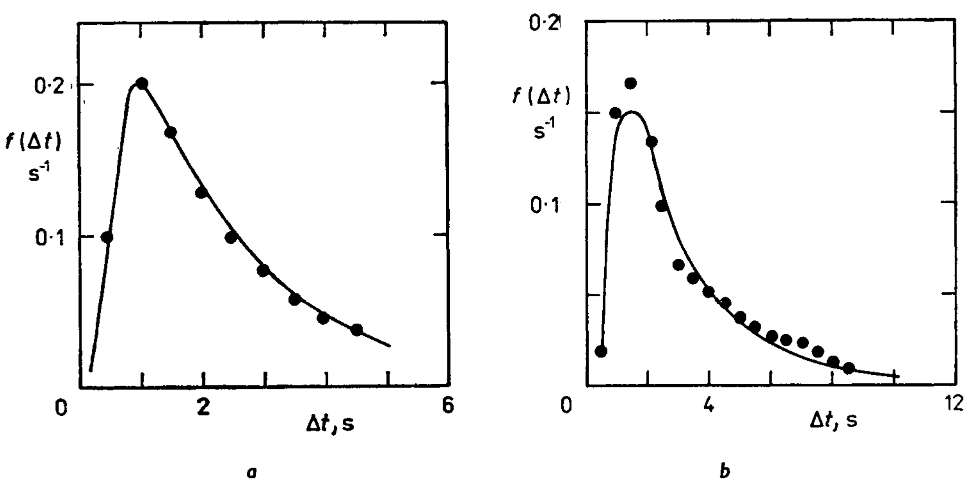

FIG. 2

Comparision of experimental data with proposed dependence for two turbine impellers and 40% solution of glycerol. *a* $n = 380\ \mathrm{min}^{-1}$, $\Delta c/d = 1{\cdot}6$; agreement according to the Pearson χ^2-test: ● experimental data, —— theoretical curve. *b* $n = 450\ \mathrm{min}^{-1}$, $\Delta c/d = 1{\cdot}4$; other conditions as in *a*; disagreement with the χ^2-test

A model equation (*4*) proposed earlier for a single impeller in the batch was selected for the description of these functions. In comparison with earlier papers[5,6] the estimates were evaluated in a more perfect way of mathematical processing. It was found out, however, that in 60% of cases we had to reject on the 5% significance level the hypothesis of the agreement of experimental estimates with the model. A better agreement was achieved only in case of a low viscosity liquid and the upper impeller and relatively large distance between both impellers. These conditions resemble the configuration with a single impeller.

Eq. (*4*) started from the assumption that there is only a single source of inertia in the system. In the studied geometrical configuration there are two sources present with complex interactions of the fluid in region between the impellers. In case of significant interactions therefore the simple earlier proposed description fails.

The experimental frequency functions are more complicated, in some cases, for instance, they seem to have an inconspicuous bimodal character when the two impellers are sufficiently remote. Examples of such a distribution are shown in Fig. 3. More complicated shapes of the frequency functions are typical primarily for the upper regions of the batch. Bimodal character of the frequency function was found also by Funamashi et al.[3] even though for other conditions and geometry of the

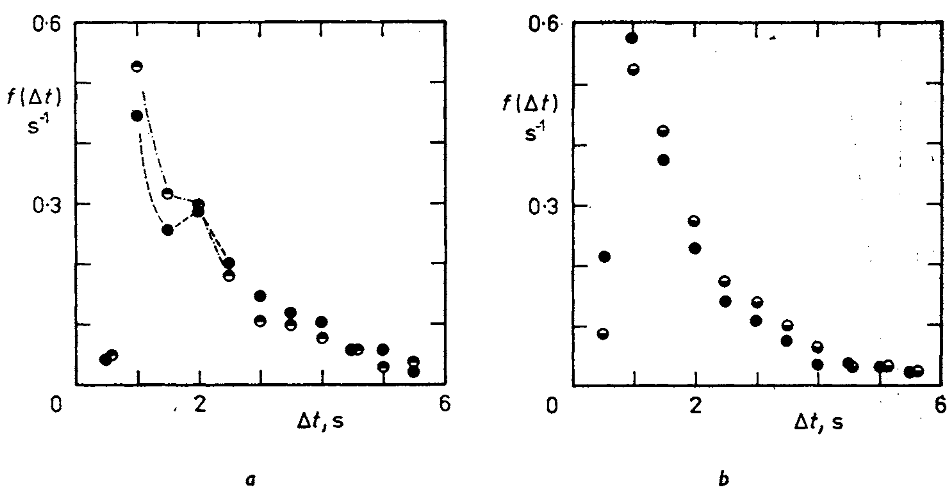

FIG. 3

Example of experimental estimate of the frequency function for six blade impeller with inclined blades above turbine impeller. *a* $n = 300\ \mathrm{min}^{-1}$, ● $\Delta c/d = 3$, ⊖ $\Delta c/d = 2\cdot5$; dotted and dash-and-dot lines connect experimental points in the vicinity of a weak maximum. *b* ⊖ $\Delta c/d = 2$; ● $\Delta c/d = 1\cdot6$; other conditions as in *a*, the second maximum was not found experimental. More adequate and more complex concepts have to be used for description of such functions.

The authors wish to thank to Dr Olga Gorbunova for careful calculations and to Dr Jiří Pechoč for setting up parts of the program in the mashine code.

LIST OF SYMBOLS

b width of baffles, m
Δc distance between impellers, m
D diameter of vessel, m
d diameter of impeller, m
E exponent of semi-log expression of the width of class interval
f frequency function of circulation times, s^{-1}
g function defined in Eq. (*4b*), s^{-1}
H height of liquid in vessel, m
h function defined in Eq. (*4a*)
I record function, V
J non-negative integer
k parameter of frequency function, s^{-1}
L number of classes of empirical distribution
M_n n-th moment of distribution, s^n
m number of passages through rotor region
N mantisse of semi-log expression of width of class interval, V
P likelihood function
q limit of resolution of recording function, V
r parameter of distribution
s standard deviation of circulation times, s
T duration of experiment, s
t instant of change, s
Δt circulation time, s
V width of class interval, s
W beginning of the first class interval, s
δ division of time axis, s
η viscosity of batch, Pa s
ϱ density of batch, kg m^{-3}
φ_n dimensionless n-th moment of distribution
ω parameter of distribution, s^{-1}

Superscripts

α passage through rotor region
β type of circulation
I passage through rotor region of lower impeller
II passage through rotor region of upper impeller

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Translated by V. Staněk.